Leveraging Phase Polynomials for Quantum Circuits Optimization

Zihan Chen*, Henry Chen*, Xiangyu Gao°, Yuwei Jin▪, Minghao Guo*,
Enhyeok Jang†, Jiakang Li*, Caitlin Chan*, Won Woo Ro†, Eddy Z. Zhang*
* Rutgers University, ° University of Washington, ▪ JPMorgan Chase, † Yonsei University

Main Contribution:

PhasePoly: A **quantum circuits optimization** framework via **Phase Polynomials.**

1. Background and Motivation

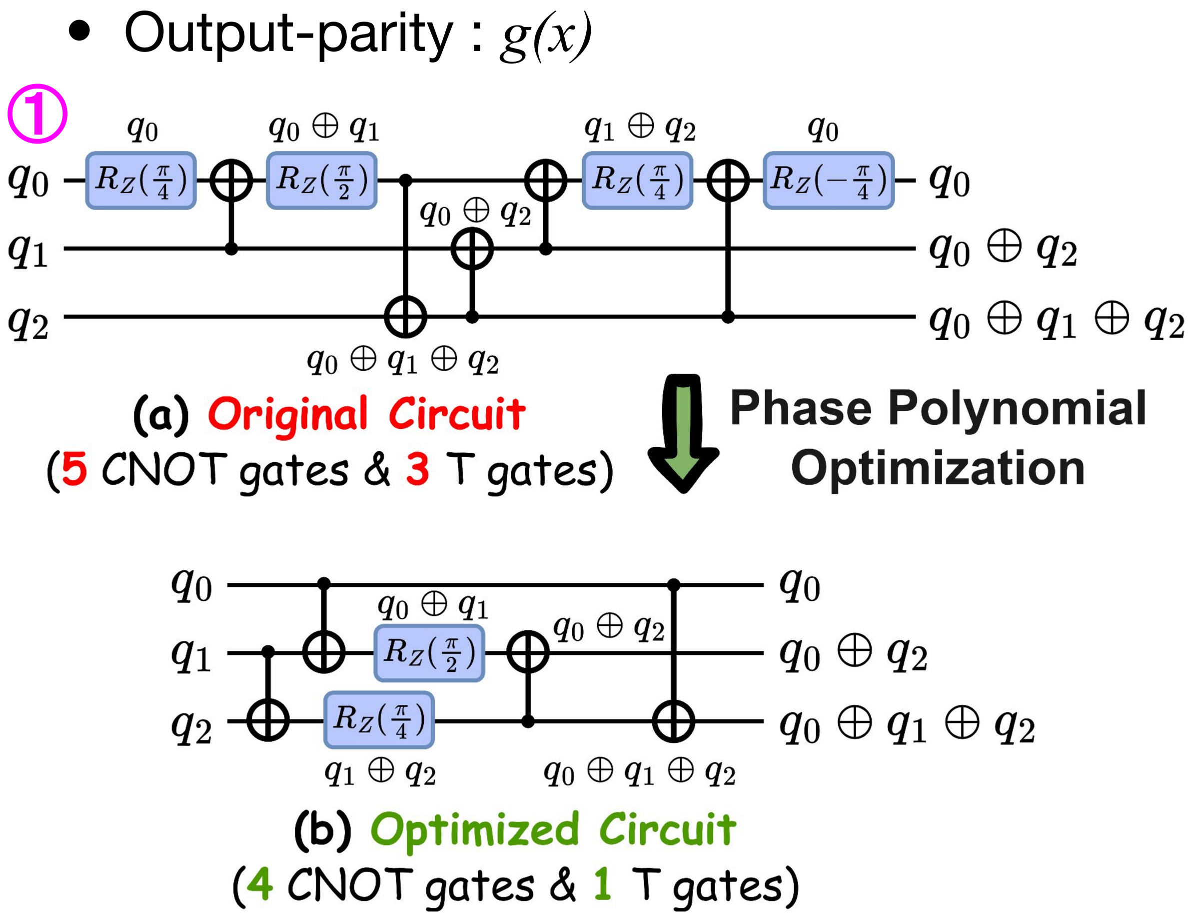
◇ Background

- We can construct a **phase polynomials** circuit using $\{\mathbf{CNOT}, \mathbf{Rz}\}$.
- Phase polynomial circuits can be represented as ***sum-over-path***[2]:

$$U|x_1,\dots,x_n\rangle = e^{ip(x_1,\dots,x_n)}|g(x_1,\dots,x_n)\rangle$$

- Phase-parity network: $p(x)$
- Output-parity : $g(x)$

①



(a) Original Circuit (5 CNOT gates & 3 T gates)

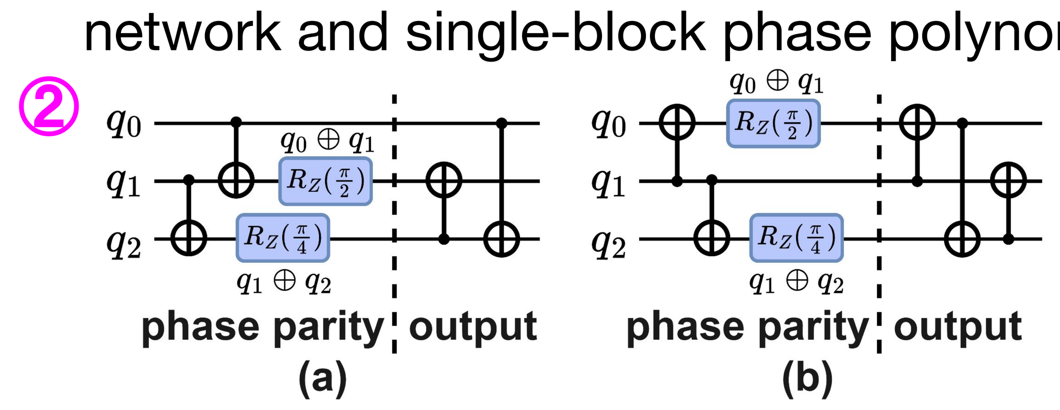
Phase Polynomial Optimization

(b) Optimized Circuit (4 CNOT gates & 1 T gates)

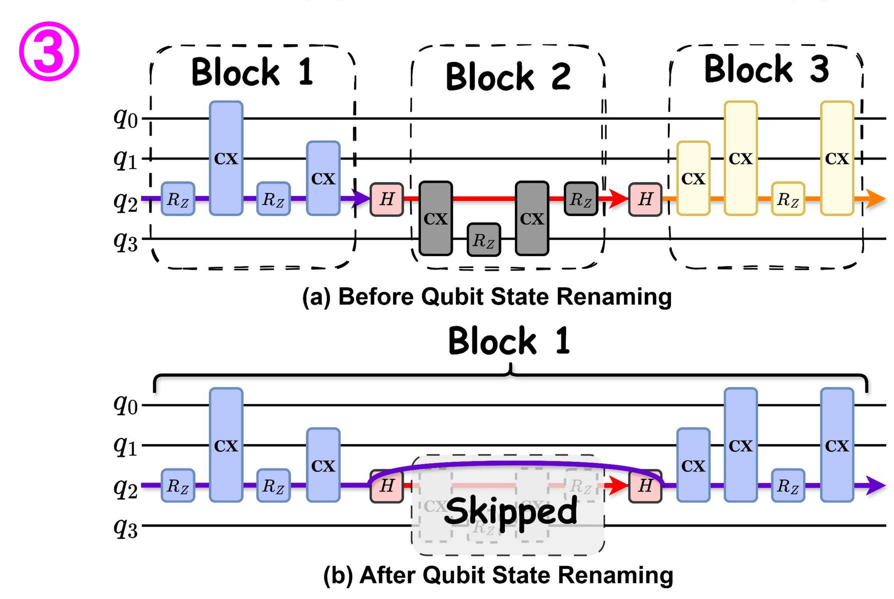
◇ Motivation

- ◇ Phase polynomials are key **building block**
  - ◇ 75% of gates are $\{\mathbf{CNOT}, \mathbf{Rz}\}$ in selected circuits
  - ◇ Commonly used in Clifford+T circuits optimization
- ◇ Current phase polynomial approaches[3]
  - ◇ **Independently** optimize phase- and output-parity network and single-block phase polynomials

②



③



- ◇ Local equivalent subcircuit rewriting approaches[6-7] struggle with **scalability**

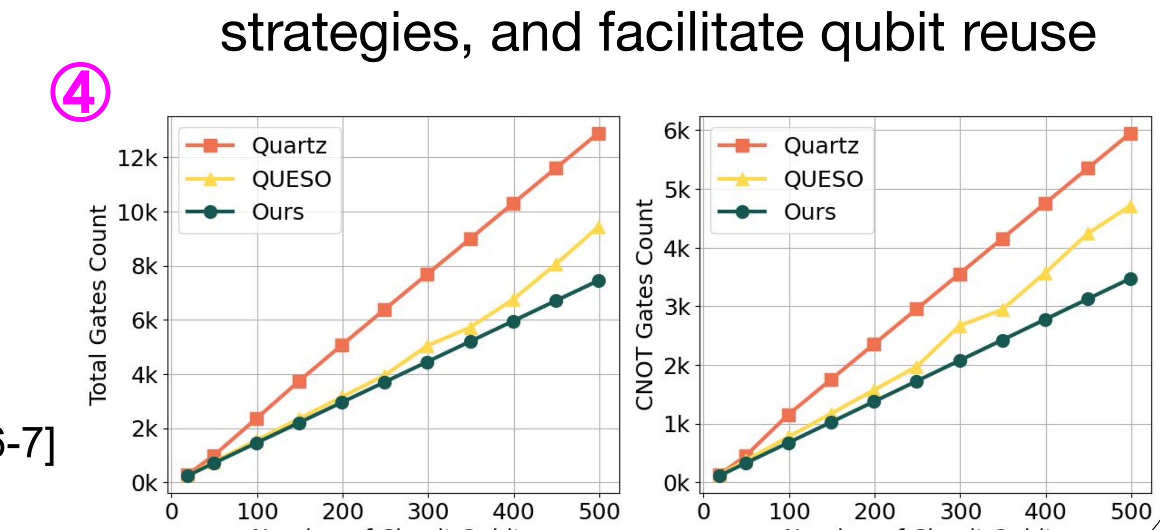
◇ Hardware-aware phase polynomials optimization

- ➔ Embed hardware constraints and qubit mapping cost into synthesis
- ➔ Maintain valid *sum-over-path* representation

◇ Fault-tolerant friendly phase polynomials optimization

- ➔ Improve full-program FTQC performance via logical-level gains
- ➔ Optimize T gate placement via phase polynomials, potentially improve magic state injection strategies, and facilitate qubit reuse

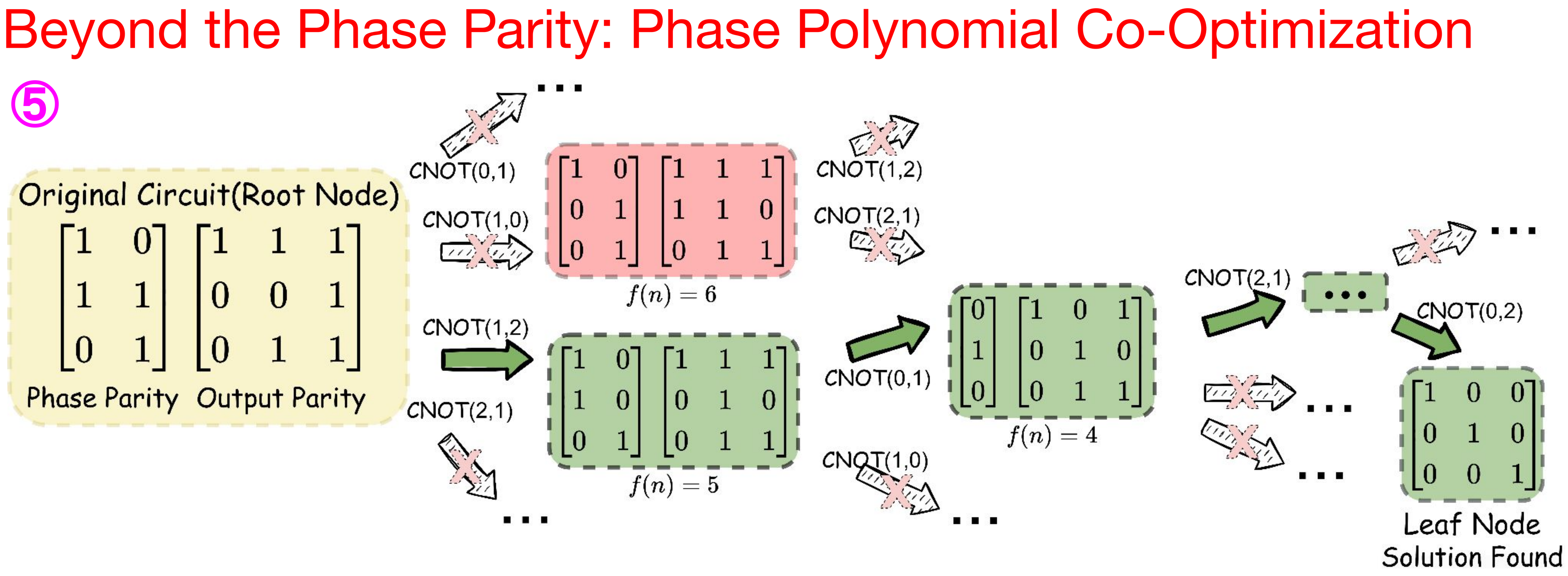
④



2. Our Approach: holistic phase polynomials optimization

◇ Beyond the Phase Parity: Phase Polynomial Co-Optimization

⑤



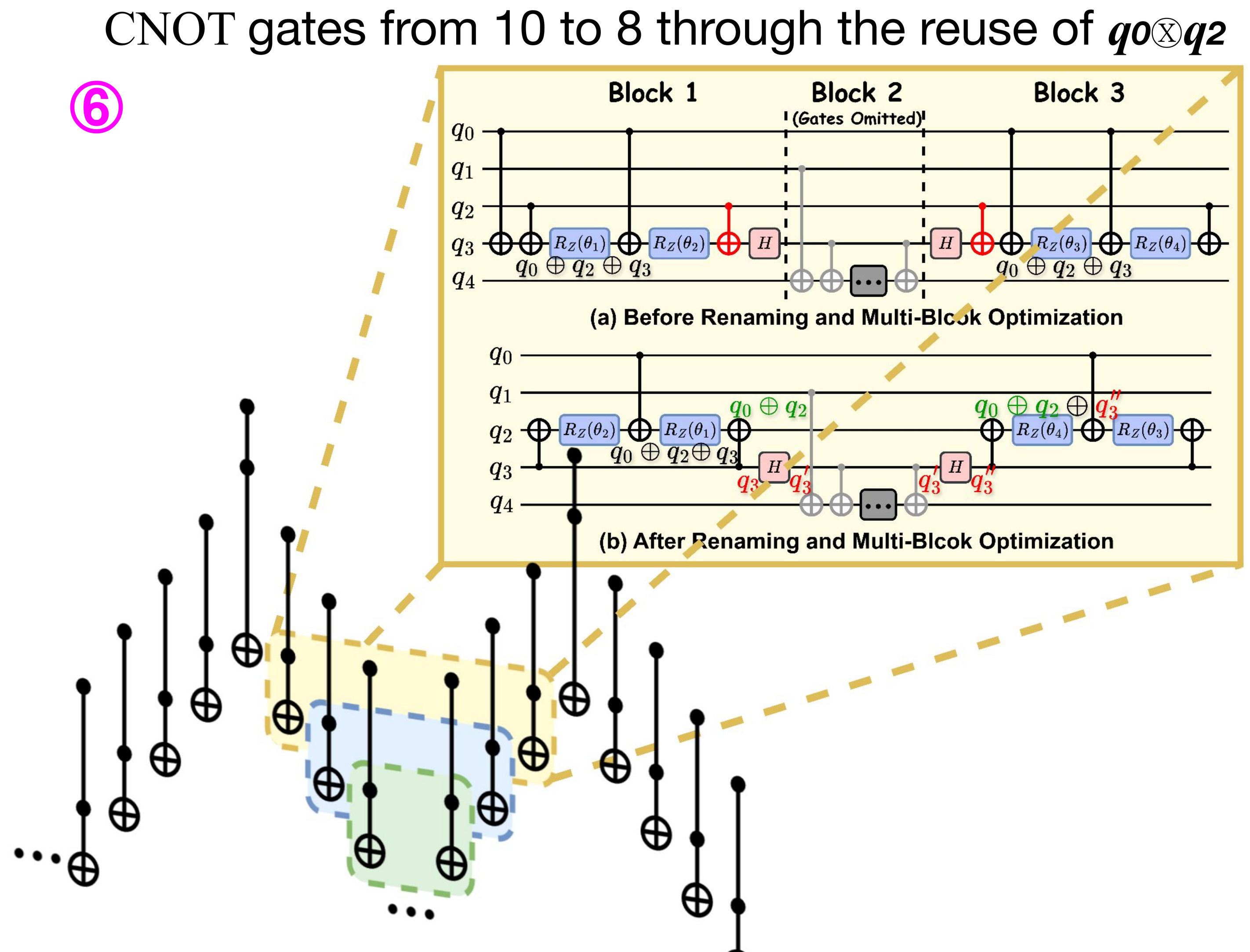
◇ Breaking the Single-Block Barrier: Cross-Block Optimization

(a) Without SSA-style cross-block intermediate representation(IR) and optimization, the term $q_0 \otimes q_2$, cannot be reused due to Hadamard gate **block barrier**

(b) After cross-block IR and optimization, three blocks are merged into a single phase polynomial region.

(c) Cross-block optimization reorders the parity network structure and reducing CNOT gates from 10 to 8 through the reuse of $q_0 \otimes q_2$

⑥



◇ Standing Alone, Working Together: Orthogonal Integration with Other Frameworks

a. **Ours alone:** strong reductions (~35% total, ~27% CNOT)

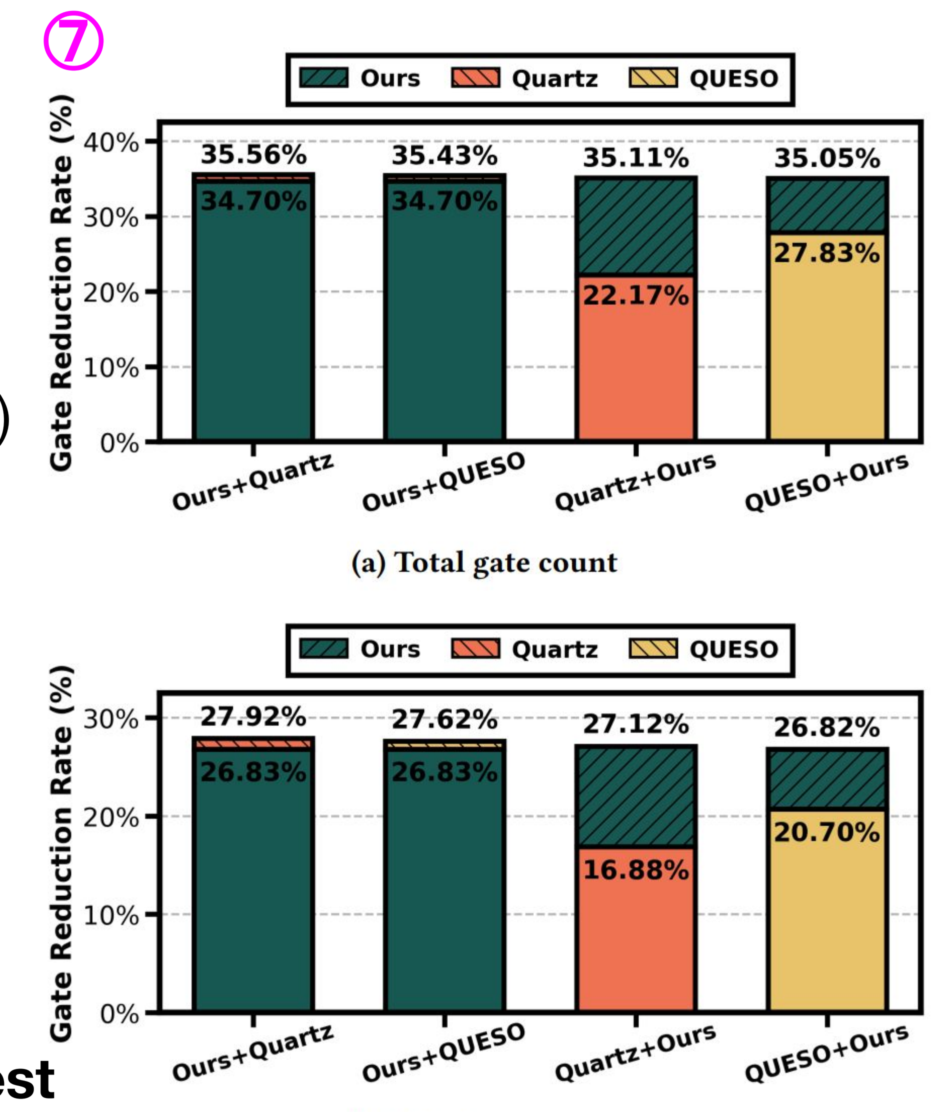
b. **Ours + Others:** only marginal gain (≤ +1%)

c. **Others + Ours:** significant results boost (+7–13% and +6–10%)

d. QUESO[7]: ✔ considers phases ☹ but still **misses a lot**

e. Quartz[6]: ✖ ignores phases; ☺ with ours, **achieves best**

⑦



(a) Total gate count

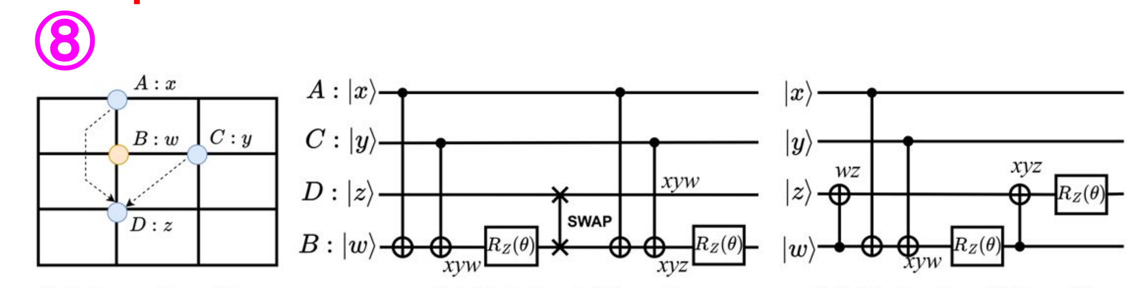
(b) CNOT gate count

◇ Results and Evaluation

- ◇ **Benchmarks**: Clifford+T benchmarks from prior work[3-7], as well as additional near-term and fault-tolerant applications.
- ◇ **Metrics**: Total and CNOT count.
- ◇ **Baselines**: GRAY-SYNTH[3] (with T gate optimizations), Quartz[6], and QUESO[7] (state-of-the-art subcircuit rewriting)
- ◇ **Verification**: Pass equivalence checking by Qiskit and MQT QCEC
- ◇ **Results**: PhasePoly outperforms both prior frameworks individually, reducing up to **50%** total gate and **48.57%** CNOT reduction. (averaging **34.70%** and **26.83%**, respectively)

◇ Extensible phase polynomials optimization

⑧



(a) Coupling Map (b) Original Circuit (c) Optimized Circuit

Reference:

[1] Chen, Zihan, et al. PhasePoly: An Optimization Framework for Phase Polynomials in Quantum Circuits. (2025).
[2] Amy, Matthew, et al. Polynomial-time T-depth optimization of Clifford+T circuits via matroid partitioning. (2014).
[3] Amy, Matthew, et al. On the controlled-NOT complexity of controlled-NOT–phase circuits. (2018).
[4] Nam, Yunseong, et al. Automated optimization of large quantum circuits with continuous parameters. (2018).
[5] Vandaele, Vivien, et al. Phase polynomials synthesis algorithms for NISQ architectures and beyond. (2022).
[6] Xu, Mingkuan, et al. Quartz: superoptimization of quantum circuits. (2022).
[7] Xu, Amanda, et al. Synthesizing quantum-circuit optimizers. (2023).